9/3 Recitation

In this recitation, you will need the commands found in the **Julia4Systems.ipynb** notebook.

1. Consider the map $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $T\Big(\begin{bmatrix} x \\ y \end{bmatrix}\Big) = \begin{bmatrix} 2x + y \\ x \\ -2y \end{bmatrix}$.

   Verify that T is a linear transformation and write down the matrix associated to T.

2. In this problem, we will verify (in an example) that the operation of matrix multiplication corresponds to function composition (as discussed in lecture). Let $S, T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$S\Big(\begin{bmatrix} x \\ y \end{bmatrix}\Big) = \begin{bmatrix} 2x + y \\ x \end{bmatrix} \text{ and } T\Big(\begin{bmatrix} x \\ y \end{bmatrix}\Big) = \begin{bmatrix} x - y \\ y \end{bmatrix}$$

   Let A be the matrix associated to S and let B be the matrix associated to T.

   (a) Determine A and B.

   (b) Compute AB.

   (c) Next compute $(S \circ T)\Big(\begin{bmatrix} x \\ y \end{bmatrix}\Big)$ and the matrix encoding $S \circ T$. Finally, verify that this is the same matrix as AB.

3. Use Julia to plot each of the systems below and determine (geometrically), which system has no solution. For each system with a unique solution, use Julia to solve the system.

$$\text{a) } \begin{aligned} 2x - 3y &= 1 \\ x + y &= 2 \end{aligned} \qquad \text{b) } \begin{aligned} 2x - 3y &= 1 \\ 4x - 6y &= 2 \end{aligned} \qquad \text{c) } \begin{aligned} 2x - 3y &= 1 \\ x - \frac{3}{2}y &= 2 \end{aligned}$$

4. In question 3, you thought about systems with two variables. Geometrically, these correspond to lines in $\mathbb{R}^2$. Now let's think about systems of equations involving 3 variables; these correspond to planes in $\mathbb{R}^3$.

   (a) Visit `https://www.geogebra.org/m/zrsx8gwe`. Here you will see the picture associated to the system

   $$\begin{aligned} x + y + z &= 1 \\ x - y + z &= 1 \end{aligned}.$$

   Rotate the picture to get a sense of what is going on. What do the solutions to this system look like geometrically? How many solutions are there? Now change the equations to see other possibilities. What do the systems

   $$\begin{aligned} x + y + z &= 1 \\ -2x - 2y - 2z &= -2 \end{aligned} \quad \text{and} \quad \begin{aligned} x + y + z &= 1 \\ x + y + z &= 2 \end{aligned}$$

   look like? How many solutions do they have?

   (b) Visit `https://www.geogebra.org/m/hb2zskvf`. Here you will see the picture associated to the system

   $$\begin{aligned} x + y + z &= 1 \\ x - y + z &= 1. \\ x + y - z &= 1 \end{aligned}$$

   How many solutions does this system have? What is it, geometrically? Next consider the systems

   $$\begin{aligned} x + y + z &= 1 \\ x - y + z &= 1 \\ -x + y - z &= -1 \end{aligned} \quad \text{and} \quad \begin{aligned} x + y + z &= 1 \\ x - y + z &= 1 \\ x - y + z &= 2 \end{aligned} \quad \text{and} \quad \begin{aligned} x + y &= 1 \\ x - y &= 1. \\ -2x + y &= 1 \end{aligned}$$

   How many solutions do these systems have?

Footnote: 3D-plotting is doable in Julia. However, it requires a fancier package than Plot() and for the purposes of question 4, geogebra is a great user-friendly tool.